Using the MRS to Evaluate Trades

We're going to see how to use the Marginal Rate of Substitution (MRS) to evaluate whether a trade will make a consumer better off, and also to identify trades that are Pareto improvements. *We're going to assume throughout that the consumer's preference is smooth and strictly quasiconvave.*

Suppose a consumer has the bundle $(\widehat{x}, \widehat{y})$. Let's consider a trade $(\Delta x, \Delta y)$ in which $\Delta x < 0$ and $\Delta y > 0$ — *i.e.*, the consumer is giving up some of the x-good and receiving some of the y-good. After the trade she will have the bundle $(\tilde{x}, \tilde{y}) = (\widehat{x}, \widehat{y}) + (\Delta x, \Delta y)$.

First suppose that the trade satisfies $\Delta y = (-\Delta x)MRS$ — *i.e.*, $-\frac{\Delta y}{\Delta x} = MRS$. This trade will not, as you might at first think, leave the consumer just as well off as at $(\widehat{x}, \widehat{y})$. She will not be indifferent between $(\widehat{x}, \widehat{y})$ and $(\tilde{x}, \tilde{y})$. The trade will make her *worse* off, as depicted in Figure 1. Indeed, any trade that has $\Delta x < 0$ and $\Delta y \leqq (-\Delta x)MRS$ will make her worse off: Δy is not large enough to compensate her for the Δx she is giving up. When $\Delta y = (-\Delta x)MRS$ we'll say that the terms of the trade are equal to the MRS, and when $\Delta y < (-\Delta x)MRS$ we'll say that the terms of the trade are less favorable than the MRS.

What about trades, still with $\Delta x < 0$, that satisfy $\Delta y > (-\Delta x)MRS$? This trade is at terms *more* favorable than the MRS. Such a trade will make the consumer better off — *if* the trade is "small enough," as in Figure 2. But if the trade is "too big," as in Figure 3, it will make the consumer worse off, even though it's at terms more favorable than the MRS. The problem, of course, is that as we make the trade larger, even at the same terms, the consumer's MRS is changing and is eventually going to be larger than at $(\widehat{x}, \widehat{y})$, so that the terms of the trade, which haven't changed, are nevertheless eventually worse than the MRS at the bundles from which increments to the trade are taking place.

However, there *is* a condition that's sufficient to guarantee that a trade at favorable terms is small enough to definitely make the consumer better off. Suppose first that the MRS *at the new bundle* $(\tilde{x}, \tilde{y})$ is equal to the terms of trade, as in Figure 4 — *i.e.*, that $\Delta y = (-\Delta x)MRS$ where the MRS is evaluated at $(\tilde{x}, \tilde{y})$, not at $(\widehat{x}, \widehat{y})$. Since the consumer's preference is strictly quasiconcave (her indifference curves are strictly convex), it must be the case that she prefers $(\tilde{x}, \tilde{y})$ to $(\widehat{x}, \widehat{y})$. But now it's also clear in the diagram that if $\Delta y > (-\Delta x)MRS$ (where again the MRS is evaluated at $(\tilde{x}, \tilde{y})$), then she prefers $(\tilde{x}, \tilde{y})$ to $(\widehat{x}, \widehat{y})$.

Summarizing for the case $\Delta x < 0$:

(∗) If $\Delta y \geqq (-\Delta x)MRS$, where MRS is evaluated at $(\widehat{x}, \widehat{y}) + (\Delta x, \Delta y)$,
then $(\widehat{x}, \widehat{y}) + (\Delta x, \Delta y)$ is strictly preferred to $(\widehat{x}, \widehat{y})$.

The same argument works as well for the case in which $\Delta y < 0$. In this case, $(\tilde{x}, \tilde{y})$ will be worse than $(\widehat{x}, \widehat{y})$ if Δx is not large enough to compensate the consumer for the Δy she is giving up, as in Figure 5, or if the trade is too large, as in Figure 6. But if $-\Delta y \leqq (\Delta x) MRS$, where the MRS is evaluated at $(\tilde{x}, \tilde{y})$, then the consumer will prefer $(\tilde{x}, \tilde{y})$ to $(\widehat{x}, \widehat{y})$, as in Figure 7. Reversing the sign on each side of this inequality gives us $(*)$, which we've now shown to hold for all $(\Delta x, \Delta y)$.

Example: Ann's and Bill's preferences are described by the utility functions

$$u_A(x_A, y_A) = x_A y_A \qquad \text{and} \qquad u_B(x_B, y_B) = y_B - \tfrac{1}{8}(4 - x_B)^2 \ .$$

Note that their marginal rates of substitution are given by

$$MRS_A = \tfrac{y_A}{x_A} \qquad \text{and} \qquad MRS_B = 1 - \tfrac{1}{4} x_B \ .$$

At the allocation $(\widehat{x}_A, \widehat{y}_A) = (4, 1)$ to Ann and $(\widehat{x}_B, \widehat{y}_B) = (0, 7)$ to Bill, we have $MRS_A = 1/4$ and $MRS_B = 1$. At the allocation $(\tilde{x}_A, \tilde{y}_A) = (3, \frac{3}{2})$ to Ann and $(\tilde{x}_B, \tilde{y}_B) = (1, \frac{13}{2})$ to Bill we have $MRS_A = 1/2$ and $MRS_B = 3/4$. Define $(\Delta x_i, \Delta y_i)$ as $(\tilde{x}_i - \widehat{x}_i,\, \tilde{y}_i - \widehat{y}_i)$ for $i = A, B$. We have $(\Delta x_A, \Delta y_A) = (-1,\, 1/2)$ and $(\Delta x_B, \Delta y_B) = (1,\, -1/2)$. Thus, the terms of trade are favorable for both Ann and Bill:

$$\Delta y_A > (-\Delta x_A) MRS_A(\widehat{x}_A, \widehat{y}_A) \qquad \text{and} \qquad \Delta y_B > (-\Delta x_B) MRS_B(\widehat{x}_B, \widehat{y}_B).$$

But as we've described above, that doesn't guarantee that either of them is made better off by the trade. However, when we evaluate MRS_A and MRS_B *after* the trade, at $(\tilde{x}_B, \tilde{y}_B)$, we still have

$$\Delta y_A = (-\Delta x_A) MRS_A(\tilde{x}_A, \tilde{y}_A) \qquad \text{and} \qquad \Delta y_B > (-\Delta x_B) MRS_B(\tilde{x}_B, \tilde{y}_B).$$

Therefore Ann strictly prefers $(\tilde{x}_A, \tilde{y}_A)$ to $(\widehat{x}_A, \widehat{y}_A)$ and Bill strictly prefers $(\tilde{x}_B, \tilde{y}_B)$ to $(\widehat{x}_B, \widehat{y}_B)$ — $((\tilde{x}_A, \tilde{y}_A), (\tilde{x}_B, \tilde{y}_B))$ is therefore a strong Pareto improvement over $((\widehat{x}_A, \widehat{y}_A), (\widehat{x}_B, \widehat{y}_B))$. Finally, note that this is still not Pareto optimal, because we still have

$$MRS_A(\tilde{x}_A, \tilde{y}_A) < MRS_B(\tilde{x}_B, \tilde{y}_B).$$

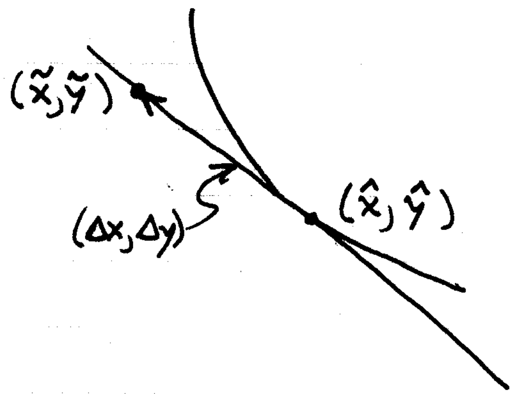

FIGURE 1

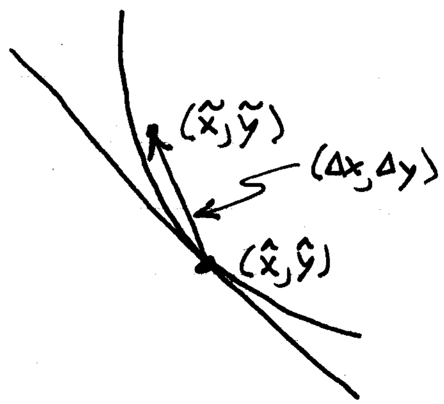

FIGURE 2

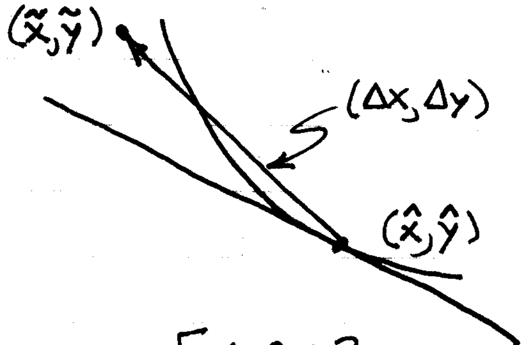

FIGURE 3

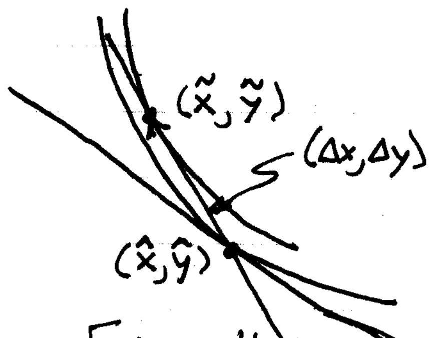

FIGURE 4

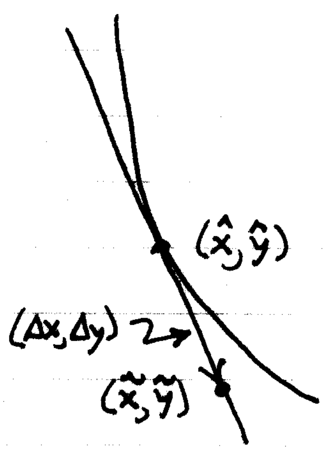

FIGURE 5

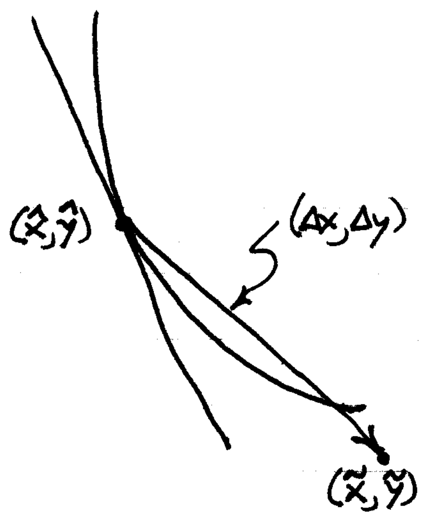

FIGURE 6

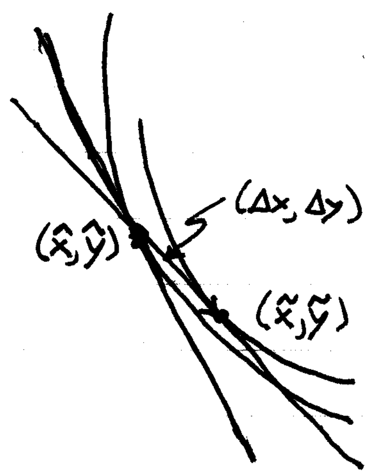

FIGURE 7